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Interference between entangled photon states in space and time

J. Visser[a] and G. Nienhuis

Huygens Laboratorium[b], Universiteit Leiden, Postbus 9504, 2300 RA Leiden, The Netherlands

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Abstract. An interference scheme is proposed for studying the spatial entanglement of twin photons. By Fourier transformation, phase and frequency matching takes the form of spatial and temporal entanglement, respectively. We use a description in space and time to show the analogy between the spatial and temporal entanglement of twin photons.

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1 Introduction

Entanglement of two separate parts of a quantum system implies that a measurement on one subsystem affects the state vector of the other one. Twin photons are often used for the study of entanglement. They are mostly created by the process of spontaneous parametric down-conversion (SPDC), by a pump pulse in a nonlinear crystal. Polarization entanglement of the photons of a twin has been studied thoroughly. In many experiments with twin photons it was found that the Bell inequalities are violated, showing that the photons of the twins are entangled [1]. It has recently been demonstrated that this entanglement is still present after the two photons have been transmitted by a metal screen with a periodic pattern of perforations, where the photons have been converted into surface plasmons [2]. In the case of polarization entanglement, the two subsystems have a two-dimensional state space. The entanglement of two particles with eigenspaces larger than the two-dimensional polarization space has been studied only recently. Vaziri et al. have demonstrated the entanglement of orbital angular momentum in an experiment with twin photons [3].

Twin photons can also be entangled in continuous degrees of freedom, such as frequency or transversal wave vector. Frequency entanglement takes the form of time entanglement after Fourier transformation to the time domain. Most theoretical descriptions of SPDC involve summations over plane-wave modes, characterized by frequency and wave vector. However, in many situations a description in terms of field operators that depend on time and transversal space more closely follows the actual life of the two photons from their creation to their detection. This can make the description more transparent. An example is the experiment by Hong et al. [4], where it is shown that the two photons of a twin arriving one in each input channel of a beam splitter at the same instant exit in the same output channel. This shows up as a destructive interference in the coincidence rate. This experiment is described in terms of integrals over wave vectors and frequencies of the emitted photons. The condition for interference that the two photons must arrive simultaneously at the beam splitter is brought out more clearly by a formulation in the domain of space and time [5]. An equivalent experiment, with two independent photons from two separate emitters, has been studied recently [6]. Franson [7] proposed an experiment where the photons of a twin can both take a short and a long path to the detector. As a consequence of temporal entanglement, there is interference between the two amplitudes where both take the long, or both take the short path. This cannot be understood as interference between single photons, only as an interference between two different histories of the photon pair. The proposed experiment has been realized by Tittel et al. [8]. The same argument holds for the experiment of Pittman et al. [9], who show that for interference to occur it is not necessary that the photons of a twin arrive at the beam splitter at the same instant. In this experiment the two photons are created with orthogonal polarization, and a polarization-dependent delay is imposed, so that the photons do not arrive at the beam splitter simultaneously. As a consequence of the polarization-dependent delay, the arrival times of the photons at the detectors generally contain information on the polarization. The temporal entanglement of the twin photons is then exploited to erase this

[a] e-mail: `jorrit@molphys.leidenuniv.nl`
[b] URL: `http://molphys.leidenuniv.nl/qo/index.html`

information, so that interference with 100% visibility is recovered. Viewed in the time domain, these experiments rely on the fact that the photons of a twin are created at the same instant of time, but where the exact time of creation of the twin is undetermined.

We point out that, apart from temporal entanglement, the photons of a twin can also be spatially entangled. After Fourier transformation to the transversal space, entanglement of the transversal wave vector takes the form of spatial entanglement. Due to the local nature of the nonlinear interaction in the crystal, the photons of a twin are created at the same instant of time and at the same position. The two-photon state is a linear superposition of states corresponding to different times of creation, which are undetermined within the duration of the pump pulse. The same holds for the location in the crystal. The photons of a twin are created at the same location in the crystal, but this location is undetermined within the pump spot on the crystal.

In Section 2 we give a general expression for the two-photon state created by SPDC. In Section 3 we propose an experiment for studying the spatial entanglement of twin photons. This experiment is the spatial equivalent of an experiment by Pittman et al. [9], which we discuss in Section 4. For both experiments we calculate the two-photon state and the coincidence detection rates. We use a description in the space and time domain, which more closely follows the life of the photons in the experiment. Another advantage is that then the analogy between temporal and spatial entanglement is much more evident.

2 Two-photon state

In the process of SPDC twin photons are created by a pump beam in a nonlinear crystal at one point and one time instant. The amplitude for the creation of a twin at time t and position $\vec{r}$ in the crystal is proportional to the positive-frequency part of the electric field of the classical pump beam $E_p(\vec{r},t)$. We assume that the pump beam and the photons of the twin have a common direction of propagation, which we call the z-direction. In the x–y-plane, or transversal plane, the crystal is much larger than the pump spot. The output plane of the crystal is located at $z=0$ and we write

$$E_p(\vec{r},t)|_{z=0} = g(t)G(\vec{\rho}), \tag{1}$$

where $g(t)$ is the pulse shape, and $G(\vec{\rho})$ the transversal profile of the pump beam. The two-dimensional vector $\vec{\rho}=(x,y)$ is the transversal coordinate.

For the basis of polarization we take the eigenvectors $\{\vec{\varepsilon}_H, \vec{\varepsilon}_V\}$ as determined by the crystal birefringence and the propagation direction of the pump beam. By using a pinhole twins are selected that propagate collinearly with the pump beam. The orientation of the crystal with respect to the propagation direction of the pump beam and polarization of the pump beam are chosen such that the amplitude for the photons of the twin to have orthogonal polarizations is much larger than the amplitude for identical polarizations: we have type-II phase matching. Because the crystal is birefringent, the speed and direction of propagation of a photon inside the crystal is polarization dependent. The resulting walk-offs can be compensated by using compensating crystals [1]. After compensation the two-photon state in the interaction picture can be written as

$$|\Psi(t)\rangle \propto \int_{-\infty}^{t} dt' g(t') \int d\vec{\rho}\, G(\vec{\rho}) \hat{a}_H^\dagger(\vec{\rho},t')\hat{a}_V^\dagger(\vec{\rho},t')|0\rangle, \tag{2}$$

where $\hat{a}_H^\dagger(\vec{\rho},t)$ and $\hat{a}_V^\dagger(\vec{\rho},t)$ create an H-polarized and a V-polarized photon, respectively, at time t, and transversal coordinate $\vec{\rho}$. The state $|0\rangle$ is the vacuum state. The expression for the two-photon state is only valid when the dispersion in the crystal can be neglected. This is the case when the spectrum of the pulse shape $g(t)$ is narrow band, and the photons of the twins are detected with a well-determined frequency, which is at the cost of the brightness of the parametric down-conversion. When these conditions are relaxed, the brightness increases, but the visibility in an interference experiment is reduced because of dispersion in the crystal [10,11]. Since we study the entanglement of the twin photons using two-photon interference, a large visibility is favored over a large brightness. The operators $\hat{a}_i^\dagger(\vec{\rho},t)$ with $i=H,V$ can be assumed to satisfy the commutation relation

$$[a_i(\vec{\rho},t), a_j^\dagger(\vec{\rho}\,',t')] = \delta_{ij}\delta(\vec{\rho}-\vec{\rho}\,')\delta(t-t'), \tag{3}$$

where $i,j=H,V$ [12]. Note that we describe the process in the domain of time t and transversal position $\vec{\rho}$, rather than the more common picture of frequency ω and transversal wave vector $\vec{\kappa}$. These pictures are related by Fourier transformations as

$$\hat{a}_i(\vec{\rho},t) = \frac{1}{(2\pi)^{3/2}} \int d\omega \int d\vec{\kappa}\; \hat{a}_i(\vec{\kappa},\omega) \exp(i\vec{\kappa}\cdot\vec{\rho} - i\omega t), \tag{4}$$

where the momentum-frequency field operators $\hat{a}_i(\vec{\kappa},\omega)$ obey the commutation relation

$$[\hat{a}_i(\vec{\kappa},\omega), \hat{a}_j^\dagger(\vec{\kappa}',\omega')] = \delta_{ij}\delta(\vec{\kappa}-\vec{\kappa}')\delta(\omega-\omega'). \tag{5}$$

Notice that the photon state can be perfectly localized in the transversal plane [13]. With the definitions

$$\tilde{G}(\vec{\kappa}) = \frac{1}{(2\pi)^2} \int d\vec{\rho}\, G(\vec{\rho}) \exp(-i\vec{\kappa}\cdot\vec{\rho}), \tag{6}$$

$$\tilde{g}(\omega,t) = \frac{1}{2\pi} \int_{\infty}^{t} dt'\; g(t') \exp(i\omega t'), \tag{7}$$

the two-photon state in equation (2) can be written as

$$|\Psi(t)\rangle \propto \int d\vec{\kappa} \int d\vec{\kappa}'\; \tilde{G}(\vec{\kappa}+\vec{\kappa}') \int d\omega \times \int d\omega'\; \tilde{g}(\omega+\omega',t)\; \hat{a}_H^\dagger(\vec{\kappa},\omega)\hat{a}_V^\dagger(\vec{\kappa}',\omega')|0\rangle. \tag{8}$$

In this expression we recognize the conservation of energy and transversal momentum, which is equivalent to the fact that the two photons are created at the same location and the same instant of time.

After propagation through an optical system, which might contain several optical elements, the photons of the twins are detected in coincidence by two detectors a and b. These detectors detect photons behind linear polarizers, and integrate over time and over the transversal location in the detector plane. A component of the positive-frequency part of the electric-field operator at the transversal coordinate $\vec{\rho}_a$ in the detection plane of detector a at time t_a is written as $\hat{E}_a^+(\vec{\rho}_a, t_a)$. For detector b we write $\hat{E}_b^+(\vec{\rho}_b, t_b)$. These operators can be expressed in terms of the annihilation operators $\hat{a}_i(\vec{\rho}, t)$, where $i = H, V$. These expressions depend on the transfer functions of the optical systems. The transversal part of the transfer function describes the evolution of the transversal profile of a light field when it propagates from the crystal plane to the detector plane. It is frequency dependent, but since a narrowband filter is used, it is justified to fix this frequency at the center frequency of the filter. As a consequence, the transfer function factorizes in a transversal and a longitudinal, or temporal part.

The amplitude where detector a detects a photon at time t_a and position $\vec{\rho}_a$, and detector b detects a photon at time t_b and position $\vec{\rho}_b$, is given by

$$A = \langle 0|\hat{E}_b^+(\vec{\rho}_b, t_b)\hat{E}_a^+(\vec{\rho}_a, t_a)|\Psi(t_a)\rangle, \tag{9}$$

where $t_b > t_a$. We shall now consider separately cases where either the spatial entanglement or the temporal entanglement is responsible for interference.

3 Spatial entanglement

3.1 Spatial interferometer

We use the interference scheme in Figure 1 to study spatial entanglement. A pump beam creates twin photons in a nonlinear crystal by the process of SPDC. The orientation of the crystal with respect to the propagation direction of the pump beam and the polarization of the pump beam are such, that the twin photons that propagate collinearly have orthogonal polarization. The walk-offs that result from the birefringence of the crystal, are compensated by compensating crystals. For simplicity they are not shown in the picture. A pinhole is used to select collinear twins. The crystal birefringence together with the direction of propagation defines a unique basis for the polarization, consisting of the unit vectors $\vec{\varepsilon}_H$ and $\vec{\varepsilon}_V$. A translation $\Delta\vec{s}_V$ in the transversal direction is imposed on V-polarized photons. This is done by propagation of the beam through a tilted birefringent crystal. Then also a time difference between H- and V-polarized photons results, introducing polarization information in the arrival time of the photons at the detectors. This polarization information can be erased by propagation through a compensating crystal with the appropriate thickness. The collinear twins fall on a 50%:50% beam splitter. In one of the output channels a translation $\Delta\vec{s}_H$ is imposed in the transversal direction on H-polarized photons. Coincidences are detected by the detectors a and b in the output channels. In front of both detectors are narrowband filters and linear polarizers set to transmit when the polarization is at an angle of 45° with respect to both $\vec{\varepsilon}_H$ and $\vec{\varepsilon}_V$. The narrowband filters have a center frequency of half the frequency of the pump beam. The detectors are bucket detectors that integrate both over the time and the transversal space. In front of detector b is also a circular aperture with radius d. For this setup the coincidence detection rate is considered as a function of the translation $\Delta\vec{s}_H$ for fixed value of $\Delta\vec{s}_V$. Now two amplitudes are relevant. The first is the amplitude that the H-polarized photon is detected by detector a and the V-polarized photon is detected by detector b. For the second amplitude it is the other way around. It is necessary that the polarizers are both at 45°, because then the information about the polarization is completely erased, and these amplitudes can interfere.

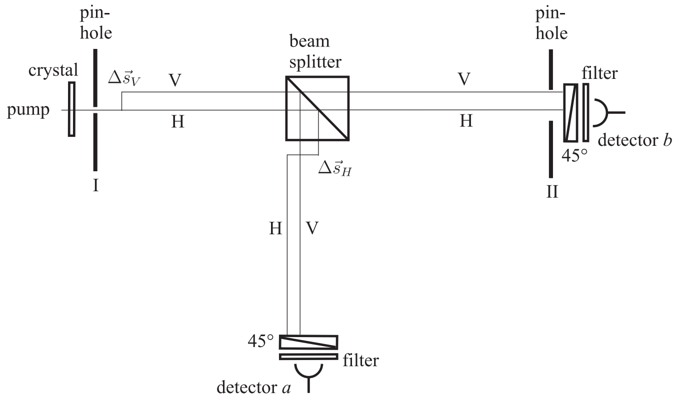

Fig. 1. Scheme of the spatial interferometer. The pump beam creates twin photons in a nonlinear crystal. Collinear twins with orthogonal polarizations are selected with a pinhole. On V-polarized photons a transversal translation $\Delta\vec{s}_V$ is imposed. Then the beam falls on a beam splitter. In one of the output channels a transversal translation $\Delta\vec{s}_H$ is imposed on H-polarized photons. The detectors a and b detect coincidences in the two output channels. In front of both detectors is a narrowband filter and a linear polarizer at 45°. There is a pinhole in front of detector b.

3.2 The coincidence detection rate

In Section 2 we argued that the transfer functions that describe the propagation from the crystal to the detector, factorize in transversal and temporal parts. In the interferometer in Figure 1 there are no optical elements that introduce any polarization dependence in the temporal properties. As a consequence the temporal part of the problem drops out, and we can write the two-photon state in equation (2) as

$$|\Psi\rangle \propto \int d\vec{\rho}\, G(\vec{\rho})\hat{a}_H^\dagger(\vec{\rho})\hat{a}_V^\dagger(\vec{\rho})|0\rangle, \tag{10}$$

where $\hat{a}^\dagger_H(\vec{\rho})$ and $\hat{a}^\dagger_V(\vec{\rho})$ create a photon at the transversal position $\vec{\rho}$ in the crystal plane, with H and V polarization, respectively. For simplicity the translation $\Delta\vec{s}_V$ is imposed at the output plane of the crystal, and the translation $\Delta\vec{s}_H$ is imposed immediately behind the output plane of the beam splitter. The detectors are both at a distance z from the crystal along the optical lines. The transfer function for free space propagation over a distance z is given by

$$h_f(\vec{\rho},\vec{\rho}\,';z) = \frac{k}{2\pi z}\exp\left[\frac{ik}{2z}\left(\vec{\rho}-\vec{\rho}\,'\right)^2\right], \tag{11}$$

where k is the wave number of the light. We consider as an example the transfer function for the propagation from the output plane of the crystal to the detection plane of detector a for a beam with H polarization. We split up the optical line in two parts with lengths z_1 and z_2, where the first part stretches from the crystal plane to the output plane of the beam splitter, and the second from the latter plane to the detector plane. The vectors $\vec{\rho}$, $\vec{\rho}\,'$, and $\vec{\rho}_a$ refer to a point in the plane of the crystal, beam splitter, and detector, respectively. Then the transfer function for the propagation from the crystal to detector a for an H-polarized beam is given by

$$\int d\vec{\rho}\,' h_f(\vec{\rho}_a,\vec{\rho}\,'+\Delta\vec{s}_H;z_2)h_f(\vec{\rho}\,',\vec{\rho};z_1) = h_f(\vec{\rho}_a,\vec{\rho}+\Delta\vec{s}_H;z), \tag{12}$$

where $z = z_1 + z_2$. We have ignored that the beam splitter changes the handedness of the basis of the transversal space. We see that for the transfer function it is not relevant in which plane the translation $\Delta\vec{s}_H$ is imposed. Now we can express the positive-frequency parts of the electric-field operators in the detector planes in terms of the annihilation operators $\hat{a}_H(\vec{\rho})$ and $\hat{a}_V(\vec{\rho})$, that annihilate a photon at the location $\vec{\rho}$ in the detector plane with H and V polarization, respectively. Because the polarizers in front of the detectors are at 45°, only photons with polarization vector $(\vec{\varepsilon}_H+\vec{\varepsilon}_V)/\sqrt{2}$ are detected. Therefore we only consider the component of the electric-field operator in the 45° direction. We have

$$\begin{aligned}\hat{E}^+_a(\vec{\rho}_a) &= \int d\vec{\rho}\,[h_f(\vec{\rho}_a,\vec{\rho}+\Delta\vec{s}_H;z)\hat{a}_H(\vec{\rho}) \\ &\quad + h_f(\vec{\rho}_a,\vec{\rho}+\Delta\vec{s}_V;z)\hat{a}_V(\vec{\rho})]\,, \\ \hat{E}^+_b(\vec{\rho}_b) &= \int d\vec{\rho}\,[h_f(\vec{\rho}_b,\vec{\rho};z)\hat{a}_H(\vec{\rho}) \\ &\quad + h_f(\vec{\rho}_b,\vec{\rho}+\Delta\vec{s}_V;z)\hat{a}_V(\vec{\rho})]\,.\end{aligned} \tag{13}$$

By using equation (10) and the commutation rules in equation (3) we find that the coincidence detection amplitude in equation (9) is given by

$$\begin{aligned}A(\vec{\rho}_a,\vec{\rho}_b) &= \langle 0|\hat{E}^+_a(\vec{\rho}_a)\hat{E}^+_b(\vec{\rho}_b)|\Psi\rangle \\ &= \int d\vec{\rho}\,G(\vec{\rho})h_f(\vec{\rho}_a,\vec{\rho}+\Delta\vec{s}_H;z) \\ &\quad\times h_f(\vec{\rho}_b,\vec{\rho}+\Delta\vec{s}_V;z) \\ &\quad+\int d\vec{\rho}\,G(\vec{\rho})h_f(\vec{\rho}_a,\vec{\rho}+\Delta\vec{s}_V;z)h_f(\vec{\rho}_b,\vec{\rho};z),\end{aligned} \tag{14}$$

where only the cross products $\propto \hat{a}_H\hat{a}_V$ from $\hat{E}^+_a\hat{E}^+_b$ contribute. The coincidence detection rate is given by

$$R = \int d\vec{\rho}_a\int_{\mathrm{II}} d\vec{\rho}_b\,|A(\vec{\rho}_a,\vec{\rho}_b)|^2, \tag{15}$$

where II indicates that the integration domain is over the opening of pinhole II. We perform the integration over $\vec{\rho}_a$ and use unitarity of the transfer function for free space propagation:

$$\int d\vec{\rho}\,h_f(\vec{\rho}_2,\vec{\rho};z)h^*_f(\vec{\rho},\vec{\rho}_1;z) = \delta\left(\vec{\rho}_2-\vec{\rho}_1\right). \tag{16}$$

We then find that

$$\begin{aligned}R = \frac{k^2d^2}{2\pi z^2}&\int d\vec{\rho}\,|G(\vec{\rho})|^2 \\ &+2\mathrm{Re}\int d\vec{\rho}\,G(\vec{\rho})G^*(\vec{\rho}+\Delta\vec{s}_V-\Delta\vec{s}_H) \\ \times&\int_{\mathrm{II}} d\vec{\rho}_b\;h_f(\vec{\rho}_b,\vec{\rho};z)h^*_f(\vec{\rho}_b,\vec{\rho}+2\Delta\vec{s}_V-\Delta\vec{s}_H;z).\end{aligned} \tag{17}$$

For the integration over $\vec{\rho}_b$ we use the property

$$\int_{\mathrm{II}} d\vec{\rho}\,'h_f(\vec{\rho}\,',\vec{\rho}+\Delta\vec{s}/2;z)h^*_f(\vec{\rho}\,',\vec{\rho}-\Delta\vec{s}/2;z) = \frac{kd}{2\pi z\|\Delta\vec{s}\|}J_1(kd\|\Delta\vec{s}\|/z)\exp\left[ik\vec{\rho}\cdot\Delta\vec{s}/z\right], \tag{18}$$

where

$$J_n(x) = \frac{1}{2\pi i^n}\int_0^{2\pi} d\phi\;\exp\left(ix\cos\phi+in\phi\right) \qquad n=0,1,2,... \tag{19}$$

are the Bessel functions of the first kind. By using this property we find that

$$\begin{aligned}R \propto \frac{1}{2}&\int d\vec{\rho}\,|G(\vec{\rho})|^2 + \frac{z}{kd\|\Delta\vec{s}\|}J_1(kd\|\Delta\vec{s}\|/z) \\ \times\,\mathrm{Re}&\int d\vec{\rho}\;\exp\left(ik\vec{\rho}\cdot\Delta\vec{s}/z\right)G(\vec{\rho}+\Delta\vec{s}/2)G^*(\vec{\rho}-\Delta\vec{s}_H/2),\end{aligned} \tag{20}$$

where $\Delta\vec{s} = \Delta\vec{s}_H - 2\Delta\vec{s}_V$. To see interference fringes, the exponential factor inside the integral must oscillate at

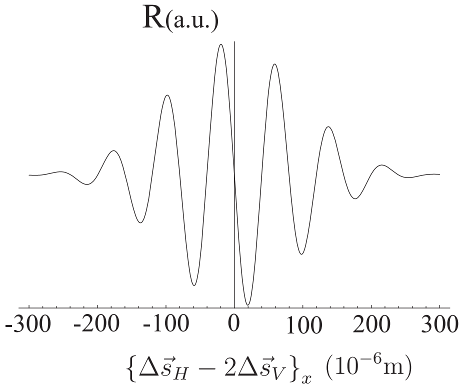

Fig. 2. Coincidence detection rate R as a function of the x-component of $\Delta\vec{s}_H - 2\Delta\vec{s}_V$ for a Gaussian pump beam tilted at 0.6°. The intensity of the pump beam drops off to $1/e$ at a distance of 3 mm from the beam axis. The wavelength of the pump beam is 400 nm. The distance from the crystal to the detectors is 2 m. For the radius of the pinhole in front of detector b we have $d = 3$ mm. The value of the x-component of $\Delta\vec{s}_V$ is fixed at 0.1 mm.

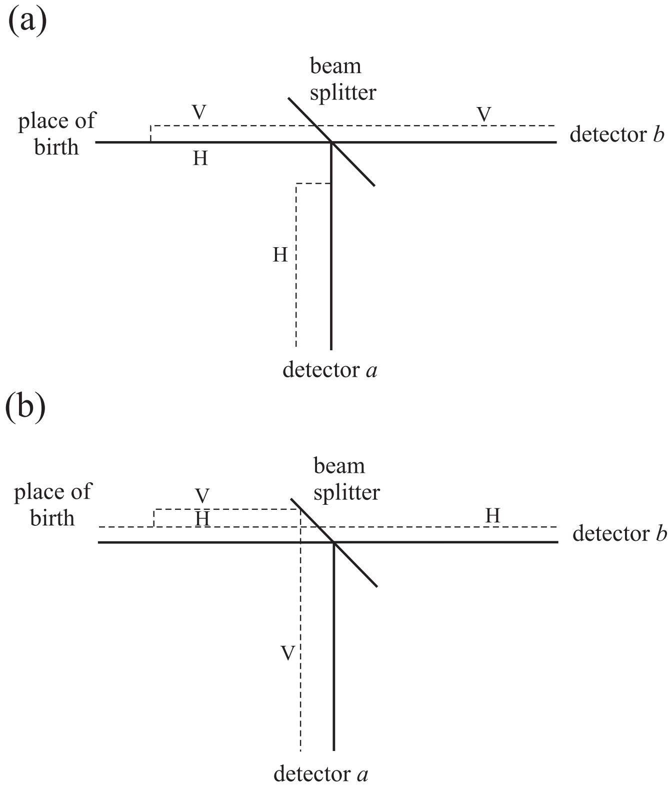

Fig. 3. The two relevant amplitudes for the case that $\Delta\vec{s}_H = 2\Delta\vec{s}_V$, where the location of birth of the twin photons for the amplitude $A(H \to a; V \to b)$ in (a) differs by $\Delta\vec{s}_V$ with respect to the location of birth for the amplitude $A(H \to b; V \to a)$ in (b). The thick lines are the optical axes of the system and the paths of the photons are dashed. We see that for both amplitudes the location at detector a at which a photon arrives, is the same, but that the polarization of the photon is different. The same holds for the photon arriving at detector b. Nevertheless, the two amplitudes are indistinguishable since the polarizers in front of both detectors erase the information about the polarization.

least once over the range of the pump spot, which puts a lower bound on $\|\Delta\vec{s}\|$. The visibility of the fringes is determined by the factor in front of the integral, which puts an upper bound on the radius d of pinhole II. As a consequence, to see interference fringes the spot size must in general not be smaller than the size of pinhole II. As an example we use a Gaussian function for the pump beam profile $G(\vec{\rho})$. In order to obtain interference fringes, we assume that the pump beam is slightly tilted, such that the propagation direction is in the x–z-plane. Then the profile of the pump beam has a phase pattern. The translations $\Delta\vec{s}_H$ and $\Delta\vec{s}_V$ are imposed in the x-direction. In Figure 2 the coincidence detection rate is given for this case as a function of the x-component of $\Delta\vec{s}_H$ for fixed value of $\Delta\vec{s}_V$. We see in Figure 2 that for $\Delta\vec{s}_H = 2\Delta\vec{s}_V$ the visibility is 100%. The reason for this can be understood by considering again the two relevant amplitudes, that we discussed at the end of Section 3.1. We first consider the amplitude $A(H \to a; V \to b)$ where the H-polarized photon is detected by detector a, and the V-polarized photon by detector b. These photons are translated by $\Delta\vec{s}_H - \Delta\vec{s}_V$ with respect to each other. For the amplitude $A(H \to b; V \to a)$ where the H- and the V-polarized photon are detected by detector b and a, respectively, the photons are translated by $\Delta\vec{s}_V$ with respect to each other. Since in front of the detector there is a linear polarizer at 45°, the information about the polarization is erased. As a consequence, these two amplitudes are indistinguishable when the polarization is concerned. For interference with maximum visibility, the two amplitudes must also be indistinguishable concerning their spatial properties. This is the case when, for the two amplitudes, the vectors over which the H- and V-polarized photon are translated with respect to each other, are identical. That is, when $\Delta\vec{s}_H = 2\Delta\vec{s}_V$, indeed. Under this condition there might still be another spatial property that distinguishes the two amplitudes. For the amplitude $A(H \to a; V \to b)$ one photon is translated by $\Delta\vec{s}_V$, and the other by $2\Delta\vec{s}_V$, while for the amplitude $A(H \to b; V \to a)$ the translations are 0 and $\Delta\vec{s}_V$. The relative translation is the same for both, but the absolute translations are different. The reason that the visibility is 100% anyway, is because of the spatial entanglement: the photons of the twins are created at the same location in the crystal, but this location itself is undetermined within the spot size of the pump beam on the crystal. Therefore, when $\Delta\vec{s}_V$ is small with respect to the spot size, the two amplitudes cannot be distinguished by a difference in the absolute translation mentioned above. This can be seen in Figure 3, where the location of birth of the twin photons for the amplitude $A(H \to a; V \to b)$ differs by $\Delta\vec{s}_V$ with respect to the location of birth for the amplitude $A(H \to b; V \to a)$. Then the two amplitudes cannot be distinguished. The width of the envelope in Figure 2 depends on the radius of the pinhole in front of detector b. A smaller pinhole radius decreases the spatial distinguishability of the detector. Because the detector is

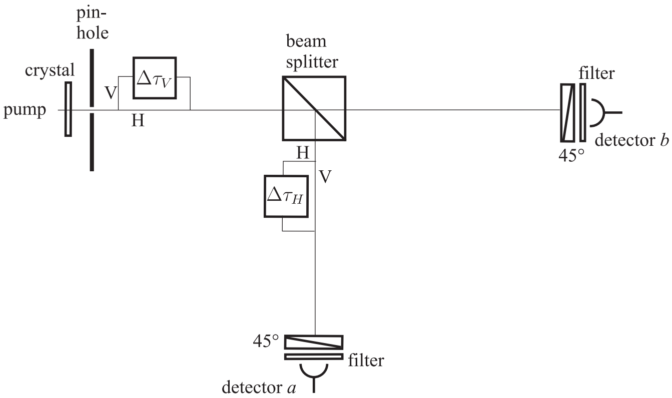

Fig. 4. Scheme of the temporal interferometer. The pump beam creates twin photons in a nonlinear crystal. A pinhole selects collinear twins with orthogonal polarization. On V-polarized photons a delay $\Delta\tau_V$ is imposed. Then the beam falls on a beam splitter. In one of the output channels a delay $\Delta\tau_H$ is imposed on H-polarized photons. Coincidences are detected by the detectors a and b in the two output channels. In front of both detectors is a narrowband filter and a linear polarizer at 45°.

then less able to obtain spatial information, the width of the envelope becomes larger.

By imposing the translation $\Delta\vec{s}_H$ on H-polarized photons the spatial information introduced by the translation $\Delta\vec{s}_V$ on the V polarization can be erased. In Figure 1 we see that the erasing is done in one of the output channels of the beam splitter. The condition for erasing is then that there must be spatial entanglement. The information can also be erased by imposing a translation $\Delta\vec{s}_H = \Delta\vec{s}_V$ on H-polarized photons before the beam splitter. Then spatial entanglement is not necessary, because before the beam splitter the collinear twins are not polarization entangled. Erasing behind the beam splitter without using spatial entanglement can be done by imposing the translation $\Delta\vec{s}_H = \Delta\vec{s}_V$ on H-polarized photons in both output channels of the beam splitter.

4 Temporal entanglement

The interference scheme that we discussed in Section 3 is the spatial analogue of the interference scheme in an experiment of Pittman et al. [9] where delays instead of transversal translations are imposed. For the spatial interferometer the spatial entanglement of the twin photons is necessary, while for the temporal interferometer it is the temporal entanglement of the twin photons. We show that the temporal interferometer of Pittman et al. is similar to the spatial interferometer. For this we calculate the coincidence detection rate as a function of the delays that are imposed. A simplified version of the experiment of Pittman et al. is given in Figure 4, which is almost exactly the same as Figure 1, except that the translations are replaced by delays. Twins photons are created with a plane wave pump by the process of SPDC in a nonlinear crystal. A pinhole selects collinear twins with orthogonal polarization. By using a birefringent crystal a delay $\Delta\tau_V$ is imposed on V-polarized photons. The collinear twins fall on a 50%:50% beam splitter. In one of the output channels of the beam splitter a delay $\Delta\tau_H$ is imposed on H-polarized photons. Coincidences are detected by the detectors a and b in the output channels. The detectors integrate over space and time. In front of both detectors are narrowband frequency filters and polarizers at an angle of 45° with respect to both $\vec{\varepsilon}_H$ and $\vec{\varepsilon}_V$. Since the basic idea of the experiment is relying on the time dependence of the field, we ignore the transversal-field distribution. This is justified under the assumption that the transversal parts of the transfer functions are independent of the polarization. As a consequence, the transversal profile does not contain polarization information. Then the transversal part of the problem does not affect the interference, and introduces only an overall factor in the coincidence detection rate. Therefore we simply write the two-photon state in equation (2) as

$$|\Psi(t)\rangle \propto \int_{-\infty}^{t} dt' g(t')\hat{a}_H^\dagger(t')\hat{a}_V^\dagger(t')|0\rangle, \tag{21}$$

where $\hat{a}_H^\dagger(t)$ and $\hat{a}_V^\dagger(t)$ create at time t an H-polarized and a V-polarized photon, respectively. The coincidence detection amplitude in equation (9) is then given by

$$A = \langle 0|\hat{E}_b^+(t_b)\hat{E}_a^+(t_a)|\Psi(t_a)\rangle, \tag{22}$$

where $t_b > t_a$.

To find the coincidence detection amplitude we express the positive-frequency part of the electric-field operator at the locations of the detectors a and b in terms of the annihilation operators $\hat{a}_H(t)$ and $\hat{a}_V(t)$. For the positive-frequency part of the electric-field operator at the location of detector a at time t_a, and detector b at time t_b, we write in the Heisenberg picture $\hat{E}_a^+(t_a)$ and $\hat{E}_b^+(t_b)$, respectively. With free propagation in one dimension, the operator $\hat{a}(z,t)$, which annihilates a photon at time t at the position z, differs from the operator $\hat{a}(0,t) \equiv \hat{a}(t)$ by a delay in time. This follows from

$$\begin{aligned} \hat{a}(z,t) &= \frac{1}{\sqrt{2\pi}} \int d\omega\ \hat{a}(\omega) \exp\left(-i\omega t + ik(\omega)z\right) \\ &= \frac{1}{\sqrt{2\pi}} \int d\omega\ \hat{a}(\omega) \exp\left[-i\omega(t - z/c)\right] \\ &= \hat{a}(0, t - z/c), \end{aligned} \tag{23}$$

where $\hat{a}(\omega)$ is the annihilation operator for a photon with frequency ω in the output plane of the crystal, and where we used the dispersion relation $k(\omega) = \omega/c$. Because the polarizers in front of the detectors are at 45°, both detectors detect photons with the polarization vector $\vec{\varepsilon} = (\vec{\varepsilon}_H + \vec{\varepsilon}_V)/\sqrt{2}$. Therefore we only consider the component of the electric-field operator in the 45° direction. The distance between the detectors and the crystal along both optical paths is z. Apart from the delay $\Delta\tau$, the time for a photon to travel from the crystal to a detector is z/c. Before being detected, the photon passes through the narrowband filter. The amplitude for the photon to reside in the filter for a duration τ is proportional to $f(\tau)$, the memory function of the filter. We have

$$\begin{aligned}\hat{E}_a^+(t_a) &= \int_0^\infty d\tau\ f(\tau)\,[\hat{a}_H(t_a - z/c - \Delta\tau_H - \tau) \\ &\quad + \hat{a}_V(t_a - z/c - \Delta\tau_V - \tau)]\,, \\ \hat{E}_b^+(t_b) &= \int_0^\infty d\tau\ f(\tau)\,[\hat{a}_H(t_b - z/c - \tau) \\ &\quad + \hat{a}_V(t_b - z/c - \Delta\tau_V - \tau)]\,. \qquad (24)\end{aligned}$$

These expressions are similar to the expressions in equation (13) that we used for the spatial interferometer. Then the expression for the coincidence detection amplitude is proportional to

$$\begin{aligned}A(t_a,t_b) = \int_{-\infty} dt\ g(t) \\ \times [f(t_b - z/c - t) f(t_a - z/c - \Delta\tau_V - t) \\ + f(t_b - z/c - \Delta\tau_V - t) f(t_a - z/c - \Delta\tau_H - t)]\,. \qquad (25)\end{aligned}$$

The upper limit of the integration in equation (25) should be the lower one of t_a and t_b. The time arguments of the filter functions in equation (25) indicate the time delay of the photons in the filters.

We consider the case that the pump beam is monochromatic with frequency ω_p, and we write

$$g(t) = \exp(-i\omega_p t). \qquad (26)$$

Then we can neglect the dispersion in the crystal and use the expression in equation (21) for the two-photon state. The frequency filters are modelled by a Lorentzian with a bandwidth α, and a center frequency that is equal to half the pump frequency ω_p, so that

$$f(\tau) = \exp(-\alpha\tau - i\omega_p\tau/2)\Theta(\tau). \qquad (27)$$

The step function $\Theta(\tau)$ makes the memory function disappear for negative τ, which reflects causality [14]. Inserting the expressions for the pump profile and the memory function of the filters into equation (25) gives the explicit expression

$$\begin{aligned}A(t_a,t_b) = \frac{i}{2\alpha}\exp[-i\omega_p(t_a + t_b - 2z/c - \Delta\tau_V)/2] \\ \times [\exp(-\alpha|t_a - t_b - \Delta\tau_V|) \\ + \exp(i\omega_p\Delta\tau_H/2)\exp(-\alpha|t_a - t_b + \Delta\tau_V - \Delta\tau_H|)]\,. \\ (28)\end{aligned}$$

The first term in equation (25) or equation (28) represents the amplitude that detector a sees the V-polarized photon, and the H-polarized photon goes to detector b. This term is maximal when $t_a - t_b = \Delta\tau_V$. The inverse situation that detector a sees the H-polarized photon and b the V-polarized one is expressed by the second term. This term is maximal when $t_a - t_b = \Delta\tau_H - \Delta\tau_V$. We assume that the detection window is large compared with the inverse bandwidth $1/\alpha$ of the filters, so that the net coincidence rate R is proportional to $\int_{-\infty}^{\infty} dt_b |A(t_a,t_b)|^2$, which gives the result

$$\begin{aligned}R \propto 1 + (1 + \alpha|\Delta\tau_H - 2\Delta\tau_V|)\cos(\omega_p\Delta\tau_H/2) \\ \times \exp(-\alpha|\Delta\tau_H - 2\Delta\tau_V|). \qquad (29)\end{aligned}$$

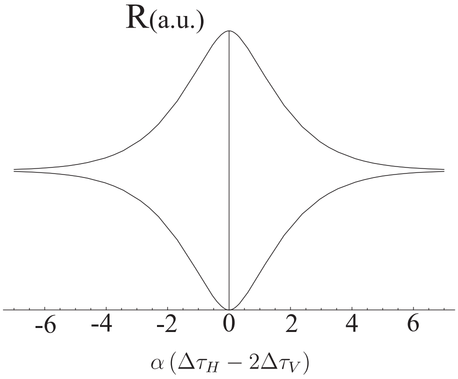

Fig. 5. Envelope of the coincidence detection rate R as a function of $\alpha(\Delta\tau_H - 2\Delta\tau_V)$ for the temporal interferometer.

In Figure 5 the envelope of the coincidence detection rate R is given as a function of the dimensionless parameter $\alpha(\Delta\tau_H - 2\Delta\tau_V)$. Here the value of $\Delta\tau_V$ is fixed, and the value of $\Delta\tau_H$ is varied. The envelope is filled with interference fringes with frequency $\omega_p/2$. With this experiment Pittman et al. show that it is not necessary for the two photons of a twin to arrive at the beam splitter at the same instant of time for interference to occur. Again two amplitudes are of importance. The first is the amplitude $A(H \to a; V \to b)$ where detector a detects the H-polarized photon, and detector b detects the V-polarized photon. For the second amplitude $A(H \to b; V \to a)$ it is the other way around. The polarizers at 45° in front of the detectors erase the information about the polarization of the detected photon, so that interference between these amplitudes can occur. Polarization information is also contained in the difference in time between the two detection events, because the photons of a twin are created at the same instant of time. For interference with 100% visibility to occur it is then necessary that the difference in arrival time at the filters is the same for both amplitudes. When $\Delta\tau_H = 2\Delta\tau_V$ this requirement is fulfilled, so that there is interference with 100% visibility, as can be seen in Figure 5. On the other hand, we see that then there is a difference of $\Delta\tau_V = \Delta\tau_H/2$ in the total travel time of the photons from the crystal to the filter for the two amplitudes. This does not destroy the interference because these amplitudes cannot be distinguished. The reason for this is, that, although the photons of a twin are created at identical times, the absolute time at which a twin is created, is undetermined, because the pump beam is monochromatic: there is temporal entanglement. We also notice that for one of the amplitudes the H-polarized photon arrives at the filter before the V-polarized photon, while it is the other way around for the other amplitude. Again the polarizers erase this difference. The interference fringes are a consequence of the fact that the pump beam changes phase under translation in time. The filters erase the information about the creation time of a photon from the observed detection time. The bandwidth α determines the width of the envelope in the coincidence detection rate when measured as a function of $\Delta\tau_H$. For the spatial interferometer the interference fringes arise because the pump beam changes phase under a translation in the transversal space, which is a consequence of the fact that the pump beam is slightly tilted. The width of the envelope in the coincidence detection rate for the spatial interferometer depends on the radius of the pinhole in front of detector b.

There is one striking difference between the description of the temporal and the spatial entanglement. The detectors integrate both over time and transversal space, but in the temporal case the ordering of the detection times of the photons is relevant, which is not the case for the spatial variant. The coincidence detection amplitude for $t_b > t_a$ is different from the one where $t_b < t_a$, as can be seen from equation (25), because the detection of a photon puts an end to the evolution of the two-photon state. On the other hand, this does not occur in the spatial case, as we see in equation (14). There is no such ordering problem with the locations of detection $\vec{\rho}_a$ and $\vec{\rho}_b$. In order to clarify the analogy between spatial and temporal entanglement we have focussed on either the spatial or the temporal aspect of the interferometer under consideration. By using the general expression for the two-photon state in equation (2) also more general experiments can be described, where both the spatial and temporal entanglement is relevant.

5 Conclusions

We have proposed an interference scheme for studying the spatial entanglement of twin photons. The spatial entanglement arises from the fact that the photons of a twin are created at the same location in the crystal, while this location itself is undetermined within the spot size of the pump beam. Expressed in transversal momentum, this spatial entanglement takes the form of transversal phase matching. The photons of the twin are created with opposite polarization and the coincidence detection signal of the two photons arises from a coherent superposition of two amplitudes. An interference structure in the coincidence detection rate occurs as a function of a polarization-dependent translation in the transversal direction. It is necessary that the photons of a twin are spatially entangled for this interference to occur.

The proposed interference scheme is the spatial analogue of the interference scheme in an experiment by Pittman et al. [9] which relies on the temporal entanglement of the twin photons. Besides that the photons of a twin are created at the same location in the crystal, they are also created at the same instant of time. The exact creation time of the twin itself is undetermined within the duration of the pump pulse. This implies that the two-photon state is entangled in time, or, equivalently, in longitudinal position along the propagation direction. In the experiment of Pittman et al. the interference occurs as a function of a polarization-dependent time delay.

We give a description of the proposed interference scheme in a form where the analogy and the difference with its temporal counterpart is best visible. We believe that this comparison elucidates the role of both types of entanglement.

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References

1. P.G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A.V. Sergienko, Y. Shih, Phys. Rev. Lett. **75**, 4337 (1995)
2. E. Altewischer, M.P. van Exter, J.P. Woerdman, Nature **418**, 304 (2002)
3. A. Vaziri, G. Weihs, A. Zeilinger, Phys. Rev. Lett. **89**, 240401 (2002)
4. C.K. Hong, Z.Y. Ou, L. Mandel, Phys. Rev. Lett. **59**, 2044 (1987)
5. Y.-H. Kim, W.P. Grice, J. Mod. Opt. **49**, 2309 (2002)
6. J. Bylander, I. Robert-Philip, I. Abram, Eur. Phys. J. D **22**, 295 (2002)
7. J.D. Franson, Phys. Rev. Lett. **62**, 2205 (1989)
8. W. Tittel, J. Brendel, B. Gisin, T. Herzog, H. Zbinden, N. Gisin, Phys. Rev. A **57**, 3229 (1998)
9. T.B. Pittman, D.V. Strekalov, A. Migdall, M.H. Rubin, A.V. Sergienko, Y.H. Shih, Phys. Rev. Lett. **77**, 1917 (1996)
10. W.P. Grice, I.A. Walmsley, Phys. Rev. A **56**, 1627 (1997)
11. J. Jan Peřina, A.V. Sergienko, B.M. Jost, B.E.A. Saleh, M.C. Teich, Phys. Rev. A **59**, 2359 (1999)
12. R. Loudon, *The Quantum Theory of Light*, 3rd edn. (Oxford Science Publications, Oxford, UK, 2000)
13. I.H. Deutsch, J.C. Garrison, Phys. Rev. A **43**, 2498 (1991)
14. J.H. Eberly, K. Wódkiewicz, J. Opt. Soc. Am. **67**, 1252 (1977)